3.7 Perpendicular Lines in the Coordinate Plane

Goals:
- Use slope to identify perpendicular lines in coordinate planes.
- Write equations of perpendicular lines.

**Example 1** Deciding Whether Lines are Perpendicular

Decide whether \( j_1 \perp j_2 \).

\[ j_1: \quad \frac{0-2}{2-(6)} = \frac{-2}{4} = \frac{-1}{2} \]

\[ j_2: \quad \frac{6-0}{5-2} = \frac{6}{3} = 2 \]

\[ \frac{-1}{2} \cdot \frac{2}{1} = -1 \quad \therefore j_1 \perp j_2 \]
Example 2 Deciding Whether Lines are Perpendicular

Decide whether the lines are perpendicular.

line s: $3x - 2y = 1$

line t: $6x + 9y = 3$

1st rewrite eqtns in slope-intercept form and identify the slope of each line.

\[
\begin{align*}
3x - 2y &= 1 \\
-3x &= -3x \\
-2y &= 1 - 3x \\
-3 &= \frac{-2y}{-2} \\
\frac{3}{2} \cdot \frac{-2}{3} &= -1 \\
&\text{so } \perp
\end{align*}
\]

\[
\begin{align*}
6x + 9y &= 3 \\
-6x &= -6x \\
9y &= \frac{3 - 6x}{9} \\
\frac{9}{9} &= \frac{3}{9} - \frac{6}{9}x \\
\frac{1}{3} - \frac{2}{3}x &= - \frac{1}{3} \\
\text{or } \frac{-3}{3}x + \frac{1}{3} &= \frac{-2}{3} \\
&\text{or } \frac{3}{2}x - \frac{1}{2} \\
&\text{m} = \frac{3}{2}
\end{align*}
\]

\[
\begin{align*}
\frac{7 - 2}{0 - 5} &= \frac{5}{-5} = -1 \\
\frac{6 - 2}{9 - 5} &= \frac{4}{4} = 1 \\
-1 \cdot 1 &= -1 \\
\text{Yes, they are } \perp
\end{align*}
\]

Checkpoint Find the slopes of the lines. Then decide whether the lines are perpendicular.

1. \[
\begin{align*}
\text{line } k_1: 6x + 2y &= 8 \\
\text{line } k_2: y &= -3x - 4 \\
m &= -3
\end{align*}
\]

\[
\begin{align*}
6x + 2y &= 8 \\
-6x &= -6x \\
\frac{2y}{2} &= \frac{8 - 6x}{2} \\
y &= 4 - 3x \\
m &= -3 \\
n &= \text{(-3)} \times 3 \times 9
\end{align*}
\]

lines are not \perp, but because slopes are the same they are parallel.
Example 3  Writing the Equation of a Perpendicular Line

Line \( r_1 \) has equation \( y = \frac{3}{x} + 5 \). Find an equation of the line \( r_2 \) that passes through \( P(3, 1) \) and is perpendicular to \( r_1 \).

The slope of \( r_1 \) is 3, so the slope of a line \( r_2 \) would be the opposite reciprocal of 3 or \(-\frac{1}{3}\). \( \frac{3 \cdot -\frac{1}{3}}{1} = -1 \)

\[ y = mx + b \]
\[ 1 = -\frac{1}{3} \cdot 3 + b \]
\[ 1 = -1 + b \]
\[ 2 = b \]

\[ y = -\frac{1}{3} x + 2 \]

Checkpoint  Find an equation of the line that passes through the given point and is perpendicular to the given line.

3. \((0, -4), y = -x\)
\[ m = -1 \]
\[ -1 \cdot 1 = -1 \]
\[ y = mx + b \]
\[ -4 = 1(0) + b \]
\[ -4 = 0 + b \]
\[ -4 = b \]

\[ y = x - 4 \]

4. \((2, -2), y = \frac{x}{4} + 10\)
\[ m = \frac{1}{4} \]
\[ \frac{1}{4} \cdot -\frac{4}{4} -1 \]
\[ \text{slope of } & \text{line would be} \]
\[ \text{be } \frac{b}{b}\]
\[ y = mx + b \]
\[-2 = \frac{1}{4}(2) + b \]
\[-2 = \frac{-8}{4} + b \]
\[+8 +8 \]
\[6 = b \]

\[ y = -4x + b \]