2.3 Deductive Reasoning

Goals:
- Use symbolic notation to represent logical statements.
- Form conclusions by applying the laws of logic to true statements.

Vocabulary:
Logical argument – an argument based on deductive reasoning, which uses facts, definitions, and accepted properties in a logical order.

**Example 1** Using Symbolic Notation

Let \( p \) be “the value of \( x \) is 7” and let \( q \) be “\( x \) is less than 10.”

a. Write \( p \rightarrow q \) in words.

b. Write \( q \rightarrow p \) in words.

c. Decide whether the biconditional statement \( p \leftrightarrow q \) is true.

**Solution**

a. If the value of \( x \) is 7, then \( x \) is less than 10.

b. If \( x \) is less than 10, then the value of \( x \) is 7.

c. The conditional statement in part (a) is true. The converse in part (b) is false. So, the biconditional statement \( p \leftrightarrow q \) is false.

**Example 2** Writing an Inverse and a Contrapositive

Let \( p \) be “my favorite TV show is on” and let \( q \) be “it is 8:00 PM.”

a. Write the contrapositive of \( p \rightarrow q \).

b. Write the inverse of \( p \rightarrow q \).

**Solution**

a. Contrapositive: \( \neg q \rightarrow \neg p \)

If it is NOT 8PM, then my fav. TV show is NOT on.

b. Inverse: \( \neg p \rightarrow \neg q \)

If my fav. TV show is NOT on, then it is NOT 8PM.
**Example 3  Using the Law of Detachment**

State whether the argument is valid.

a. If Roger gets a part-time job, then he will buy a new bicycle. Roger buys a new bicycle. So, Roger got a part-time job.

b. If two angles are vertical angles, then they are congruent. \( \angle 1 \) and \( \angle 2 \) are vertical angles. So, \( \angle 1 \) and \( \angle 2 \) are congruent.

**Solution**

a. This logical argument implies that because Roger bought a new bicycle, he **got a part-time job**. The argument is **invalid** use of the Law of Detachment.

b. The statement \( p \to q \) is **true** and the hypothesis \( p \) is **true**. So, you can conclude that the conclusion \( q \) is **true**. The argument is **valid** use of the Law of Detachment.

5. State whether the following argument is valid. If two adjacent angles form a straight angle, then the angles are supplementary. \( \angle 1 \) and \( \angle 2 \) are supplementary. So, you can conclude that \( \angle 1 \) and \( \angle 2 \) are adjacent.

Invalid we need to know that 2 adjacent angles form a straight angle.
**LAW OF SYLLOGISM**

If $p \rightarrow q$ and $q \rightarrow r$ are true conditional statements, then

\[ p \rightarrow r. \]

**Example 4  Using the Law of Syllogism**

Write some conditional statements that can be made from the following true statements using the Law of Syllogism.

1. If a cat is the largest of all cats, then it can weigh 650 pounds.
2. If a cat lives in a pride, then it is a lion.
3. If a cat weighs 650 pounds, then it is a tiger.
4. If a cat is a tiger, then it hunts alone.
5. If a cat is a lion, then it can weigh 400 pounds.

**Ex 1:** statements 1 and 3
If a cat is the largest of all cats, then it is a tiger.

**Ex 2:** statements 2 and 5
If a cat lives in a pride, then it can weigh 400 lbs.

**Ex 3:** statements 3 and 4
If a cat weighs 650 lbs, then it hunts alone.