2.2 Definitions and Biconditional Statements

Goals:
- Recognize and use definitions.
- Recognize and use biconditional statements.

Vocabulary:
- Perpendicular lines – two lines that intersect to form a right angle (\( \perp \))
- Line perpendicular to a plane – a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.
- Biconditional statement – statement that contains the phrase “if and only if” (iff)

Example 1: Using Definitions

 Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. \( \angle KJL \) and \( \angle KJL \) are complementary.

b. \( \overline{KL} \) and \( \overline{LJ} \) are perpendicular.

c. \( \angle MKJ \) is a right angle.

![Diagram showing angles and lines](image)

Check your work using the diagram in Example 1 to decide whether the statement is true. Explain your answer using the definitions you have learned.

a. True - complementary: add up to 90° (36° + 54° = 90°)

b. False - perpendicular form 90° angle (\( \angle MLJ = 34° \))

c. True - right is 90° (\( \angle MKJ = 90° \) it is vertical to \( \perp NKL \))
Conditional Statement:
If an angle measures 180 degrees then it is a straight angle.

Converse:
If an angle is a straight angle then it measures 180 degrees.

<table>
<thead>
<tr>
<th>Example 2</th>
<th>Rewriting a Biconditional Statement</th>
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<tr>
<td>An angle is a straight angle if and only if its measure is 180°.</td>
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Example 3 | Analyzing a Biconditional Statement |
<table>
<thead>
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<tr>
<td>Consider the following statement: x = 2 if and only if 3x + 5x = 10x - 2x.</td>
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<tr>
<td>a. Is this a biconditional statement? Yes</td>
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<td>b. Is the statement true? Yes</td>
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<td>Solution</td>
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<td>a. The statement is biconditional because it contains the phrase if and only if.</td>
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<td>b. The statement can be rewritten as the following statement and its converse.</td>
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<tr>
<td>Conditional statement: If ( x = 2 ), then ( 3x + 5x = 10x - 2x ).</td>
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<tr>
<td>Converse: If ( 3x + 5x = 10x - 2x ), then ( x = 2 ).</td>
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<tr>
<td>The first statement is True. The second statement is False. So, the biconditional statement is False.</td>
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Notes continue
a. Converse: If $x = 1$, then $\sqrt{x} = 1$
Converse is True

Biconditional: $\sqrt{x} = 1$ if and only if $x = 1$.

b. Converse: If two angles are congruent, then they are vertical angles.
Converse is False
Counterexample:

![Counterexample Diagram]

**Checkpoint** Complete the following exercises.

3. Rewrite the following biconditional statement as a conditional statement and its converse.
   Two angles are supplementary if and only if the sum of their measures is $180^\circ$.
   If 2 angles are supplementary, then the sum of their measures is $180^\circ$.
   If the sum of the measures of 2 angles is $180^\circ$, then the angles are supplementary.

4. Consider the following statement: Two segments are congruent if and only if they have the same length.
   a. Is the statement biconditional? Yes (if and only if)
   b. Is the statement true or false? True

If 2 segments are congruent, then they have the same length.
If 2 segments have the same length, then they are congruent.