2.2 Definitions and Biconditional Statements

Goals:

- Recognize and use definitions.
- Recognize and use biconditional statements.

Vocabulary:

Perpendicular lines – two lines that intersect to form a right angle (⊥)

Line perpendicular to a plane – a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.

Biconditional statement – statement that contains the phrase "if and only if" (iff)

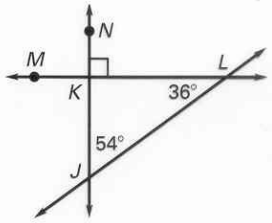
Example 1 ***Using Definitions***

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\angle KLJ$ and $\angle KJL$ are complementary.

b. $\overleftrightarrow{KL}$ and $\overleftrightarrow{LJ}$ are perpendicular.

c. $\angle MKJ$ is a right angle.



a. True - complementary: add up to 90° (36 + 54 = 90)

b. False - perpendicular form 90° angle ($\angle MLJ = 36°$)

c. True - right is 90° ($\angle MKJ = 90°$ it is vertical to $\angle NKL$)

Checkpoint **Use the diagram in Example 1 to decide whether the statement is true. Explain your answer using the definitions you have learned.**

1. $\angle KJL$ is an acute angle.	**2.** Point N is in the interior of $\angle KLJ$.
True acute is between 0-90° $\angle KJL = 54°$	False interior is inside point N is outside $\angle KLJ$.

Example 2 *Rewriting a Biconditional Statement*

Rewrite the following biconditional statement as a conditional statement and its converse.

An angle is a straight angle if and only if its measure is 180°.

Conditional Statement:

If an angle measures 180 degrees then it is a straight angle.

Converse:

If an angle is a straight angle then it measures 180 degrees.

Example 3 *Analyzing a Biconditional Statement*

Consider the following statement: $x = 2$ if and only if $3x + 5x = 10x - 2x$.

a. Is this a biconditional statement? **b.** Is the statement true?

yes

Solution

a. The statement is biconditional because it contains the phrase if and only if.

Are there any values other than $x = 2$ that make the equation true?

b. The statement can be rewritten as the following statement and its converse.

Conditional statement: If $x = 2$, then $3x + 5x = 10x - 2x$.

Converse: If $3x + 5x = 10x - 2x$, then $x = 2$.

The first statement is True. The second statement is False. So, the biconditional statement is False.

notes continue

Example 4 ***Writing a Biconditional Statement***

Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.

a. If $\sqrt{x} = 1$, then $x = 1$.

b. If two angles are vertical angles, then they are congruent.

a. Converse: If x = 1, then $\sqrt{x} = 1$

Converse is True

Biconditional: $\sqrt{x} = 1$ if and only if $x = 1$.

b. Converse: If two angles are congruent, then they are vertical angles.

Converse is False

Counterexample:

Checkpoint **Complete the following exercises.**

3. Rewrite the following biconditional statement as a conditional statement and its converse.

Two angles are supplementary if and only if the sum of their measures is 180°.

If 2 angles are supplementary, then the sum of their measures is 180°.

If the sum of the measures of 2 angles is 180°, then the angles are supplementary.

4. Consider the following statement: Two segments are congruent if and only if they have the same length.

a. Is the statement biconditional? yes (if and only if)

b. Is the statement *true* or *false*? True

If 2 segments are congruent, then they have the same length.

If 2 segments have the same length, then they are congruent.