5.1 Perpendiculars and Bisectors

Goals:

- Use properties of perpendicular bisectors.
- Use properties of angle bisectors to identify equal distances.

Vocabulary:

Perpendicular bisector – segment, ray, line, or plane that is perpendicular to a segment at its midpoint

Equidistant from two points – a point is equidistant from two points if its distance from each point is the same

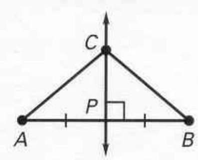
Distance from a point to a line – the length of the perpendicular segment from the point to the line

Equidistant form two lines – a point is equidistant from two lines when the point is the same distance form one line as it is from another line.

THEOREM 5.1: PERPENDICULAR BISECTOR THEOREM

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

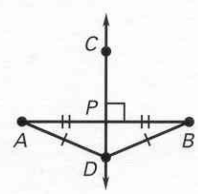
If $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$, then $CA = CB$.



THEOREM 5.2: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

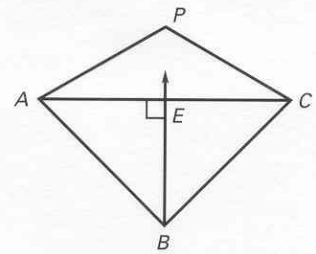
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $DA = DB$, then D lies on the perpendicular bisector of $\overline{AB}$.



Example 1 *Using Perpendicular Bisectors*

In the diagram shown, $\overrightarrow{BE}$ is the perpendicular bisector of $\overline{AC}$.



a. What segment lengths are equal?

b. $\overline{AP} \cong \overline{CP}$. What can you conclude about point P?

Solution

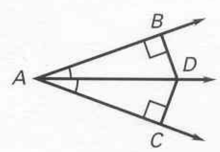
a. Because $\overrightarrow{BE}$ bisects $\overline{AC}$, AE = CE.

Because B is on the perpendicular bisector of $\overline{AC}$, you can use the Perpendicular Bisector Theorem to conclude that AB = BC.

b. Because $\overline{AP} \cong \overline{CP}$, AP = CP. Using the Converse of the Perpendicular Bisector Theorem, you can conclude that P lies on $\overrightarrow{BE}$.

THEOREM 5.3: ANGLE BISECTOR THEOREM

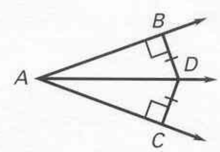
If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



If $m\angle$ BAD = $m\angle$ CAD, then $DB = DC$.

THEOREM 5.4: CONVERSE OF THE ANGLE BISECTOR THEOREM

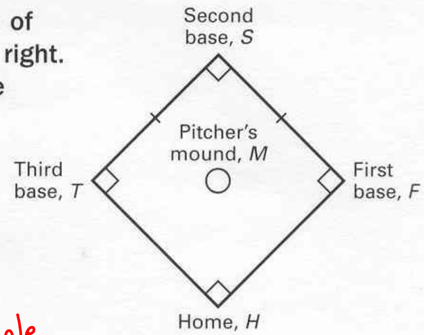
If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.



If DB = DC, then $m\angle BAD = m\angle CAD$.

Example 2 *Using Angle Bisectors*

Baseball Field Use the diagram of the baseball infield shown at the right. What can you conclude about the measure of $\angle SHF$?



Solution

From the diagram, you know that point S is in the interior of $\angle THF$ and $ST =$ SF.

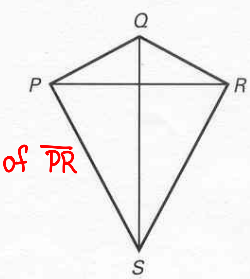
From the Converse of the Angle Bisector Theorem, you know that S lies on the angle bisector of $\angle$THF. An angle bisector divides an angle into two congruent angles, each of which has half the measure of the original angle, so

$$m\angle SHF = \frac{90^\circ}{2} = 45^\circ.$$

Answer The measure of $\angle SHF$ is 45°.

Checkpoint Complete the following exercises.

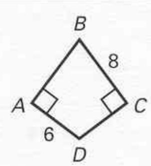
1. In the diagram, $\overline{PQ} \cong \overline{RQ}$. What conclusion can you make about point Q? Can you conclude that S is on the perpendicular bisector of $\overline{PR}$? Explain.



Q is on the perpendicular bisector of $\overline{PR}$

No - we don't know if PS = RS

2. In the diagram, D is on the bisector of $\angle ABC$. What is DC? Explain.



AD = CD and since AD = 6

CD = 6